Force distribution and contact network analysis of sheared dense suspensions

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Formative Formulation, University of Cambridge
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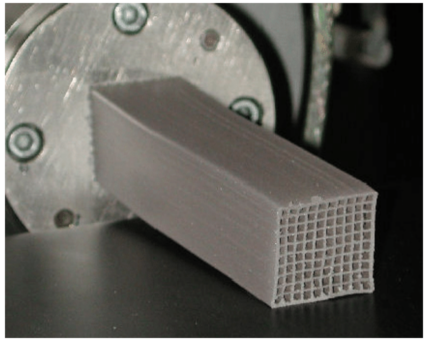
Dense suspensions of particles

Chocolate

Ceramics

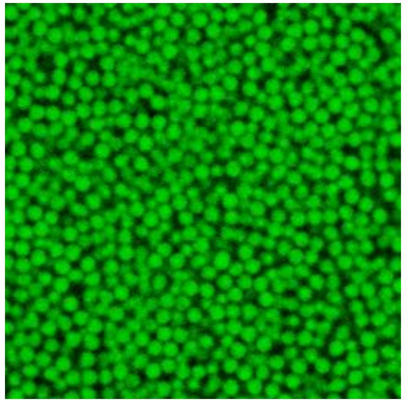


Paints

Drilling muds

Cements

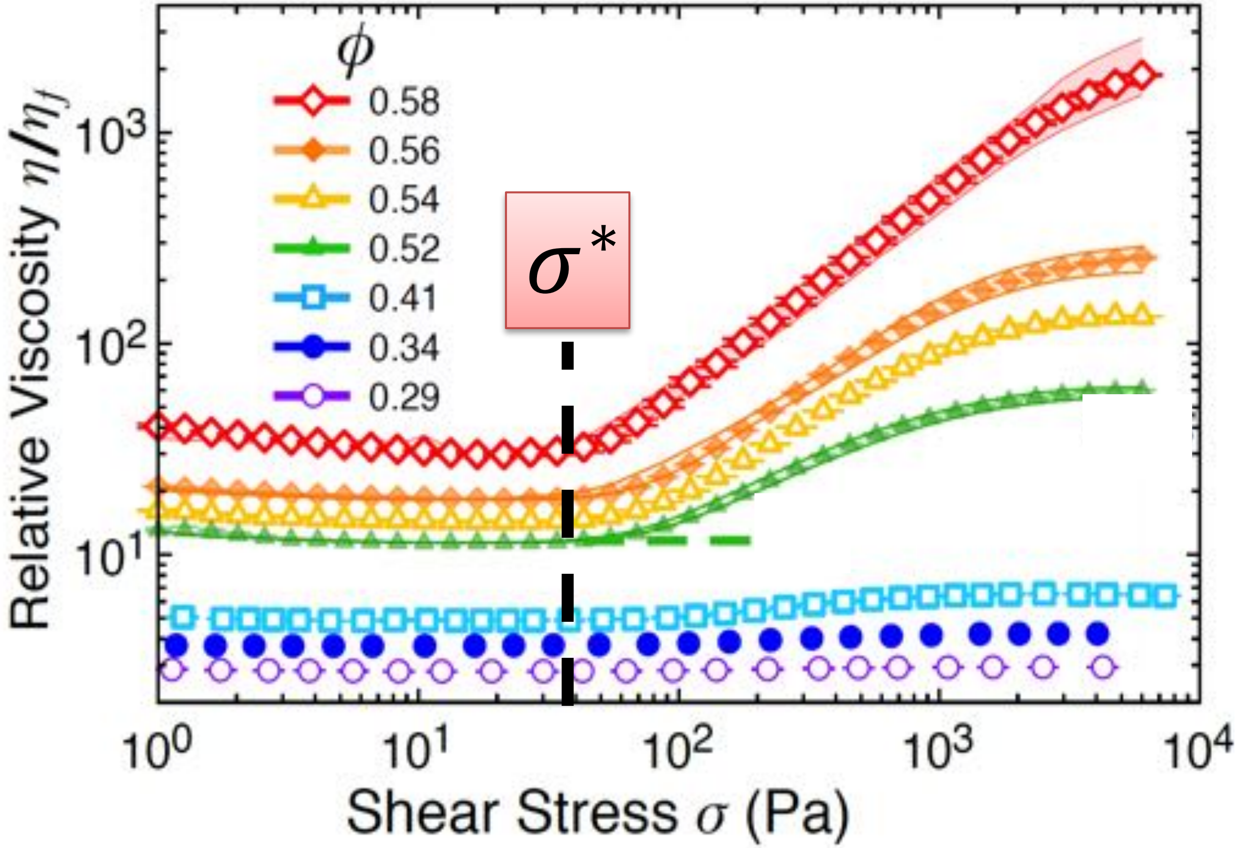
Shear thickening of model systems



2 μm Silica spheres in
Glycerol+water

$$\phi = \frac{V_{part}}{V_{total}}$$

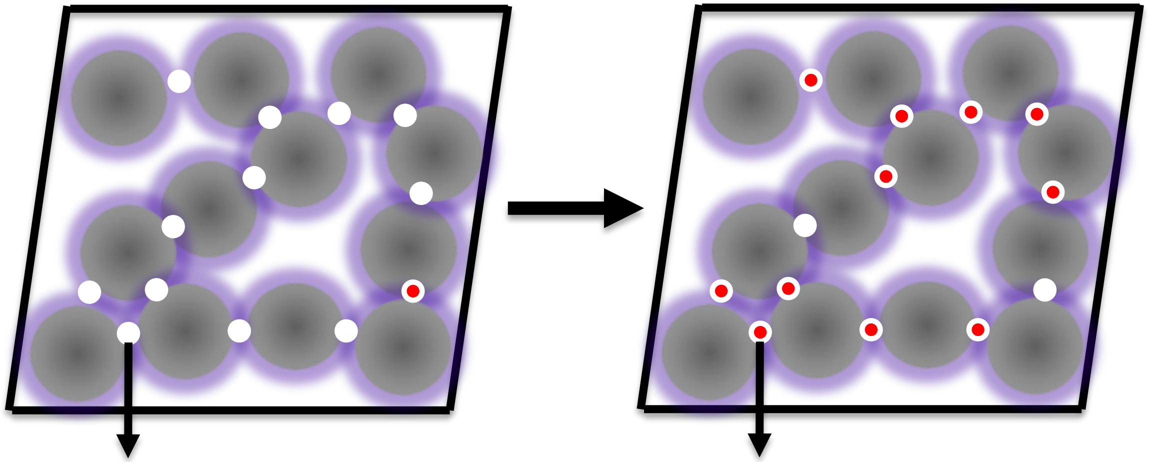
Royer, Blair, Hudson PRL 2016



Universality of shear thickening in colloidal and non-Brownian model systems.
– Guy, Hermes, Poon PRL 2015

Mechanism of shear thickening

Stress scale $\sigma^* = \frac{F^*}{d^2}$ → Repulsive force scale



$\sigma < \sigma^*$

$\sigma > \sigma^*$

Low Viscosity

High Viscosity

Repulsions prevent contact

Frictional contact

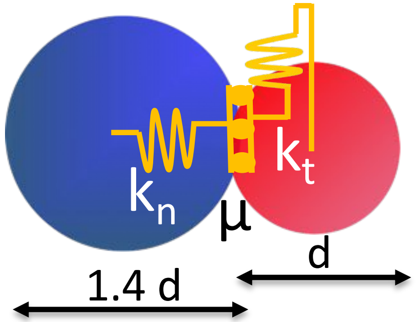
[1] Fernandez et al., PRL 2013, [2] Seto, Mari, Morris, PRL 2013, J. Rheol. 2014, [3] Wyart and Cates, PRL 2014 [4] Guy, Hermes, Poon, PRL 2015, [5] Lin et al, PRL 2015, [6] Royer, Blair, Hudson, PRL 2016, [7] Clavaud et al PNAS 2017, [8] Comtet et al, Nat. Comm. 2017

Discrete Element Method Simulations

Ness and Sun, PRE 2015

Non Brownian particles

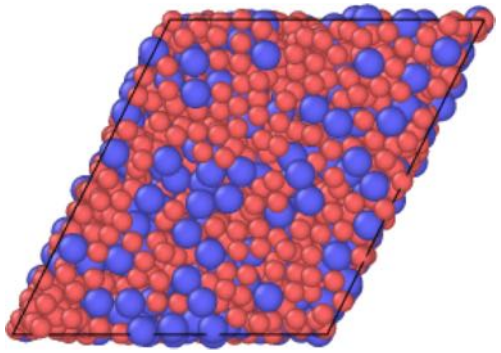
Linear (Hooke) spring force



Coulomb criterion

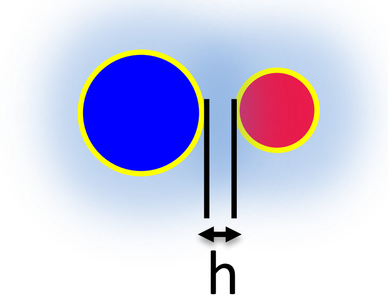
$F_t \leq \mu F_n$

Imposed shear rate $\dot{\gamma}$



LAMMPS (Sandia)

Hydrodynamic forces

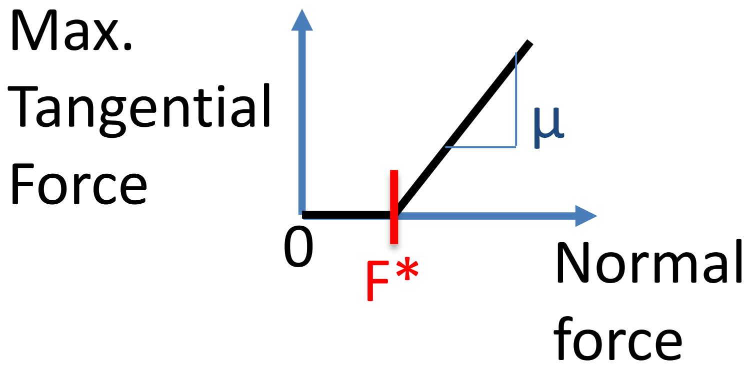


Regularised lubrication

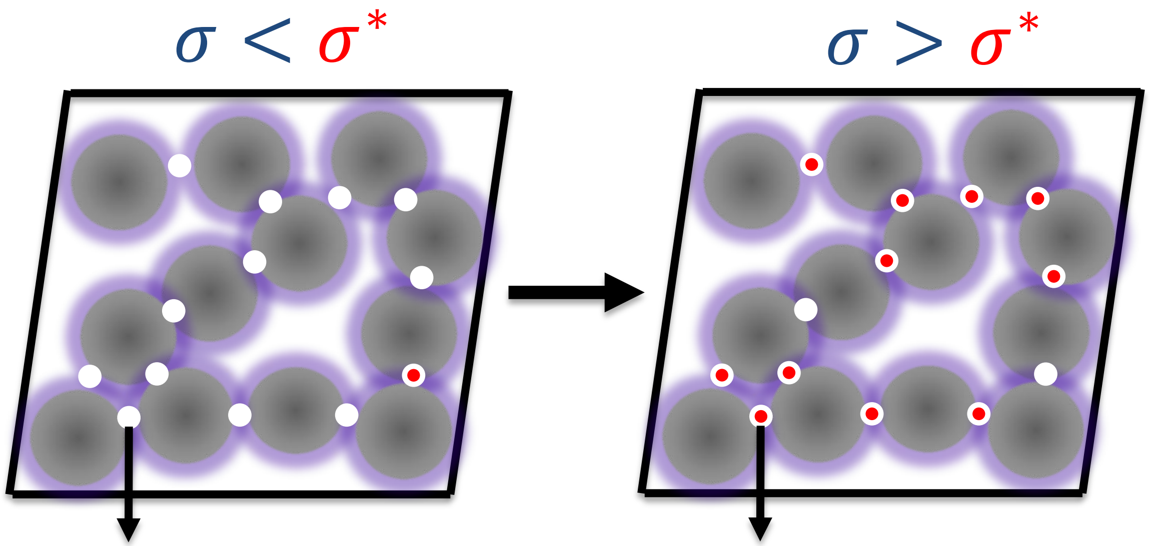
Stokes drag

Model of shear thickening

Critical load model



Stress scale $\sigma^* = \dfrac{F^*}{d^2}$ → Critical load

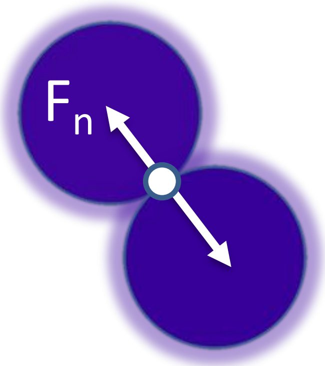


Frictionless particle contact

Frictional contact

[1] Mari, Seto, Denn, Morris, PRL 2013, J. Rheol. 2014 [2] Wyart and Cates, PRL 2014
[3] Ness CJ, and Sun J, Soft Matter 2016 [4] Clavaud et al PNAS 2017, [5] Comtet et al, Nat. Comm. 2017

Viscosity divergence at low stress $\sigma < \sigma^*$

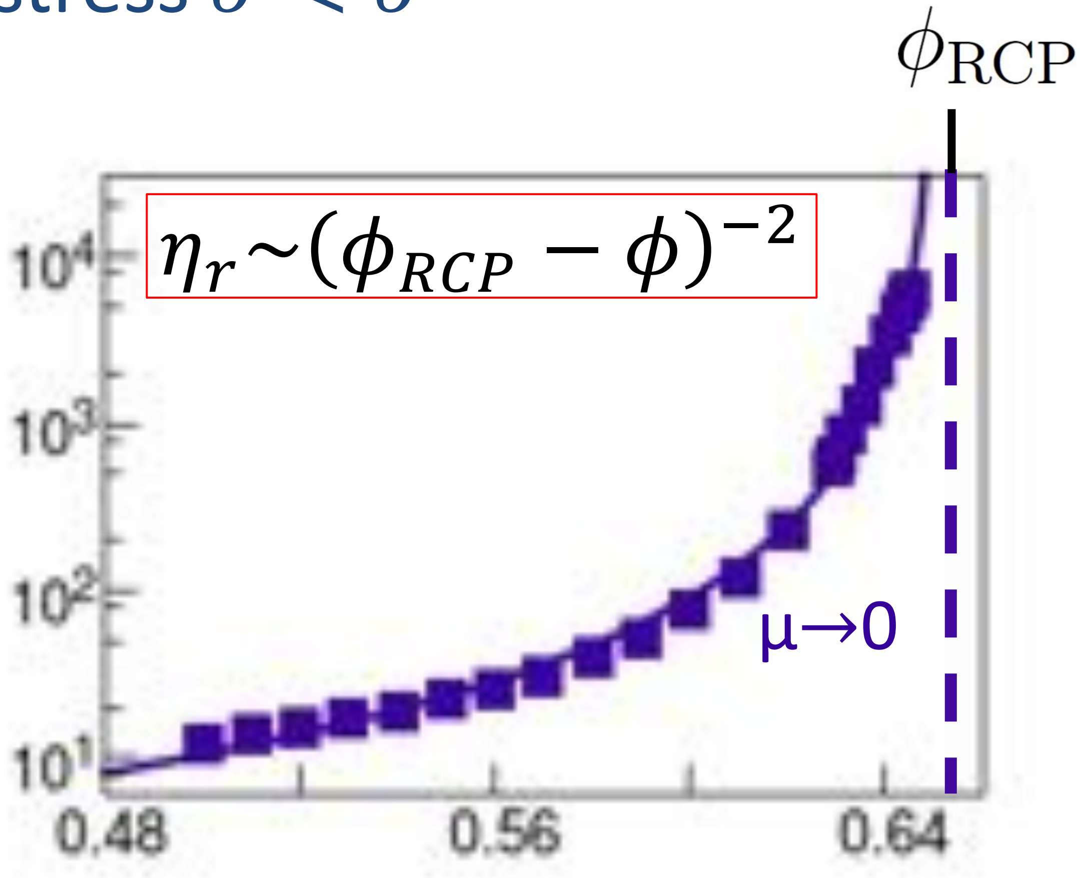


frictionless particles

$F_t \rightarrow 0$

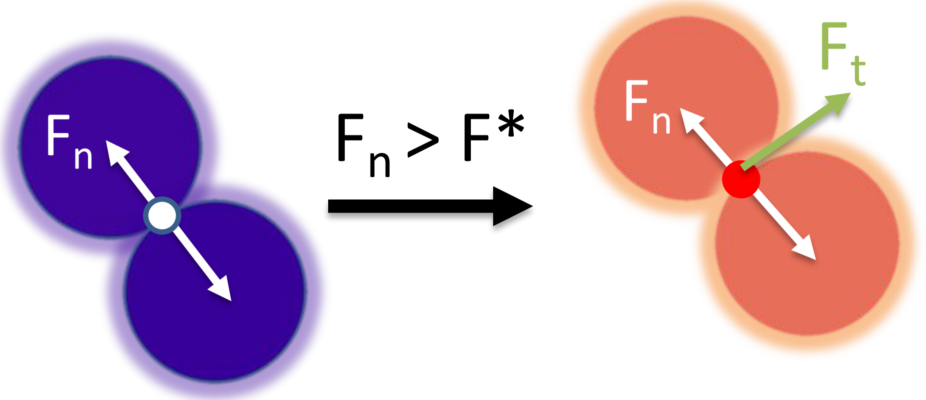
Relative Viscosity

$$\eta_r = \frac{\eta}{\eta_f}$$


$$\eta_r \sim (\phi_{RCP} - \phi)^{-2}$$

Volume Fraction ϕ

Viscosity divergence at high stress $\sigma > \sigma^*$

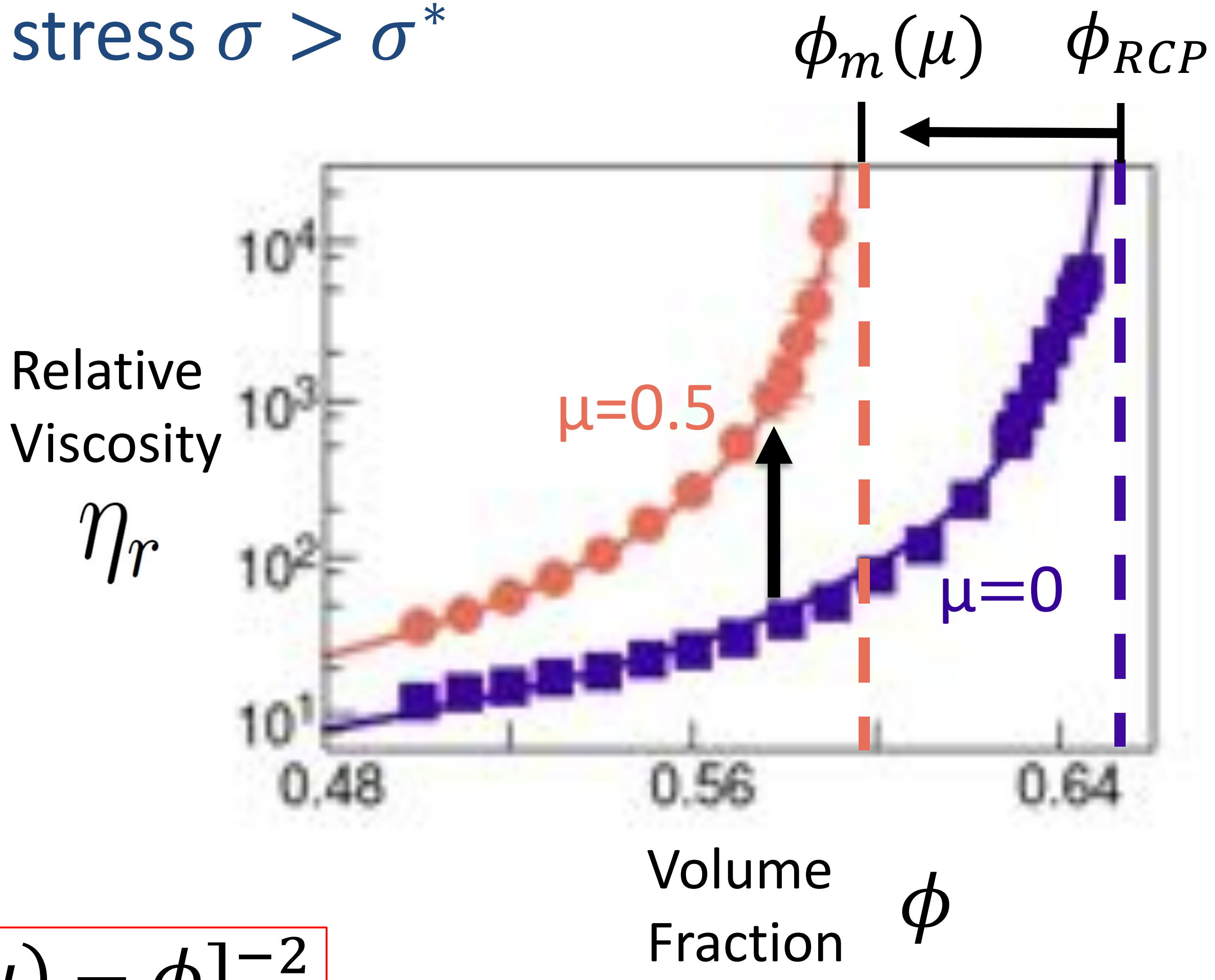


frictionless particles

$F_t \to 0$

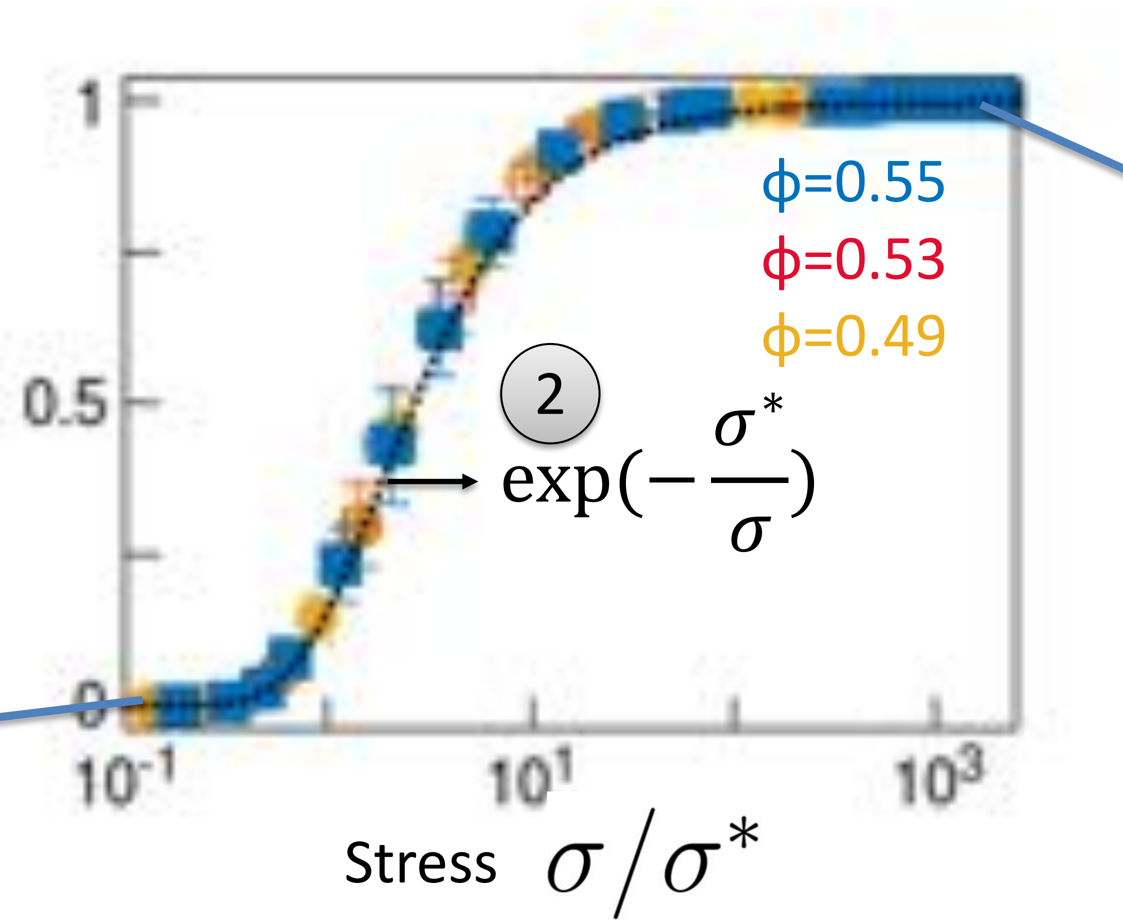
frictional particles

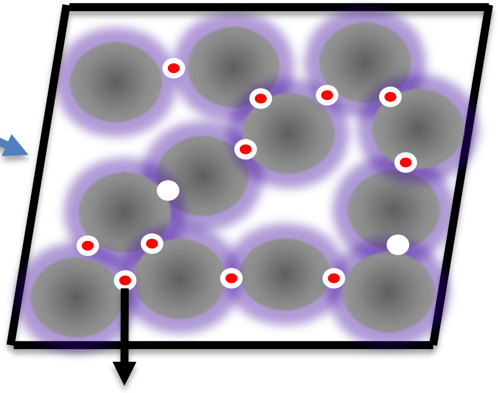
$F_t \leq \mu F_n$

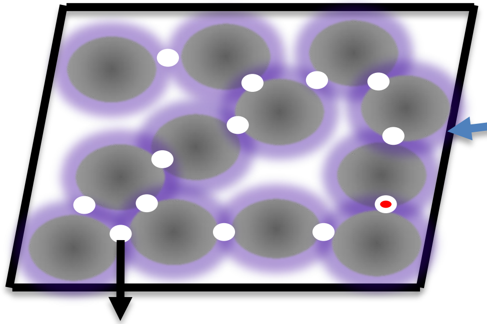


① $$\boxed{\eta_r \sim [\phi_m(\mu) - \phi]^{-2}}$$

Fraction of frictional contacts $f(\sigma)$



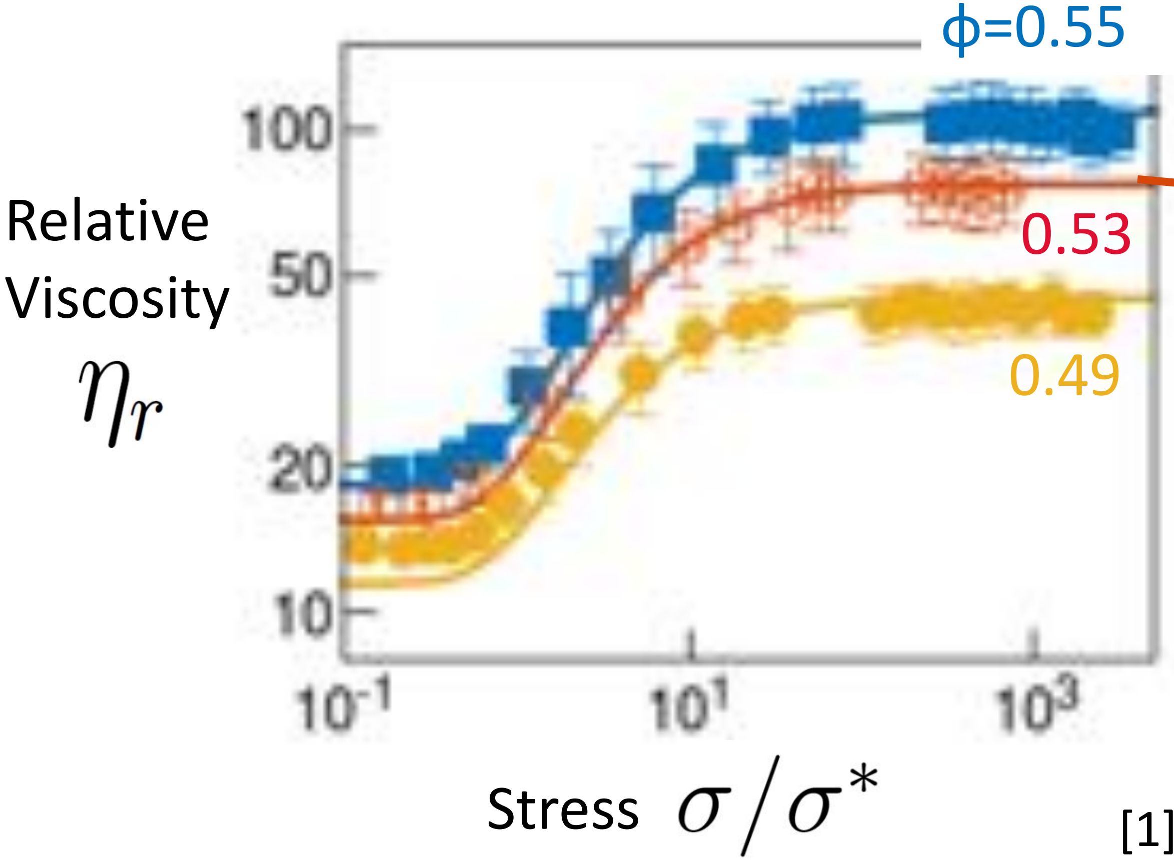




[1] Mari, Seto, Denn and Morris, PRL 2013
[2] Wyart and Cates, PRL 2014

Theory for shear thickening

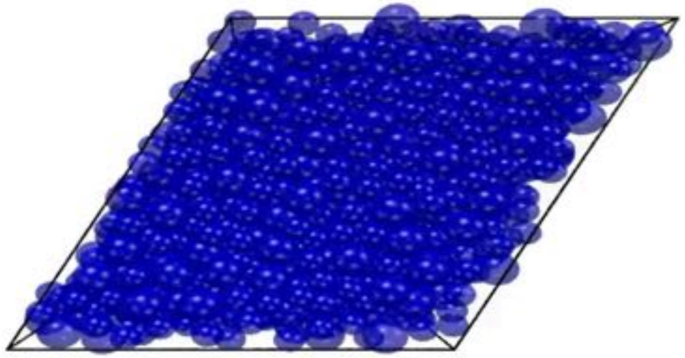
③ $$\boxed{\phi_J(\sigma) = f(\sigma)\ \phi_m + \big(1 - f(\sigma)\big)\phi_{RCP}}$$



$$\boxed{\eta_r(\sigma) \sim [\phi_J(\sigma) - \phi]^{-2}}$$

[1] Wyart and Cates, PRL 2014
[2] Singh, Mari, Denn and Morris, J. Rheol 2018

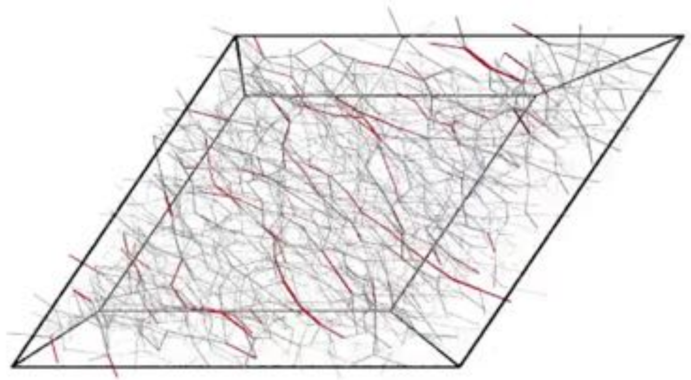
DEM simulations for a microscopic understanding

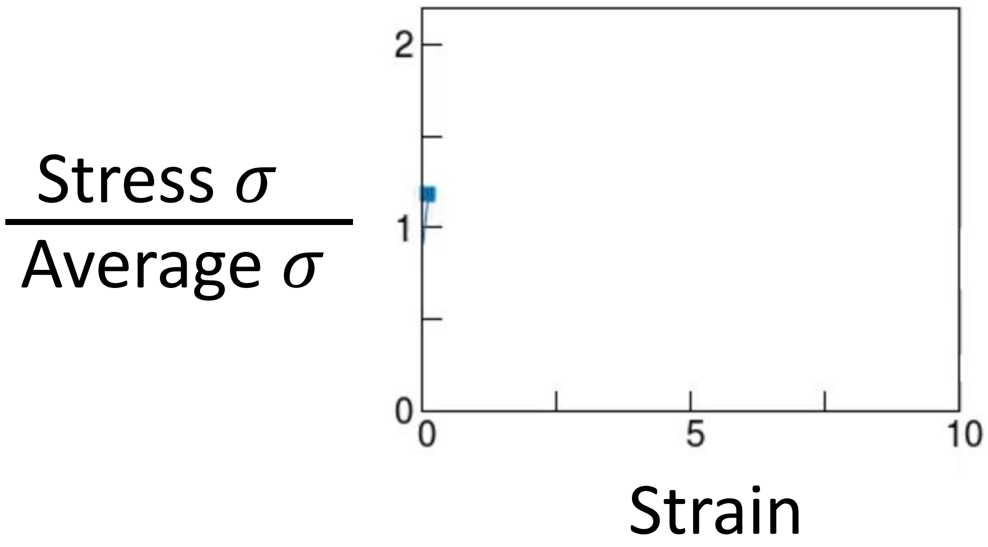


$\mu = 0.5$

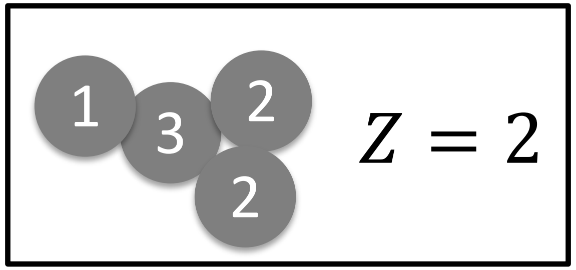
$\phi_m - \phi = 0.005$

Contact Force Network



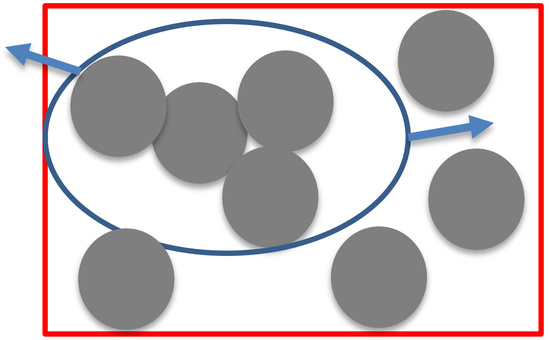


Coordination number Z and viscosity divergence



$Z = 2$

Soft Modes $\Delta Z = Z_m - Z$



$$\eta_r \sim \Delta Z^{-p}$$

For $\mu \rightarrow 0$

$p = 2.1$ (2D)
$p = 2.7$ (3D)

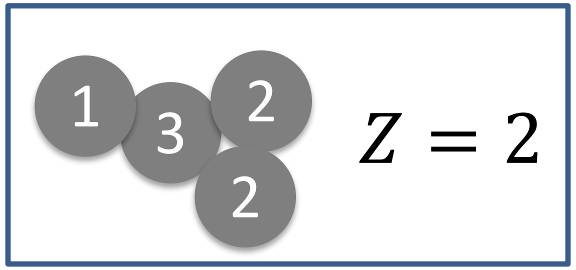
$$(Z_m(\mu) - Z) \sim (\phi_m(\mu) - \phi)$$

[1]

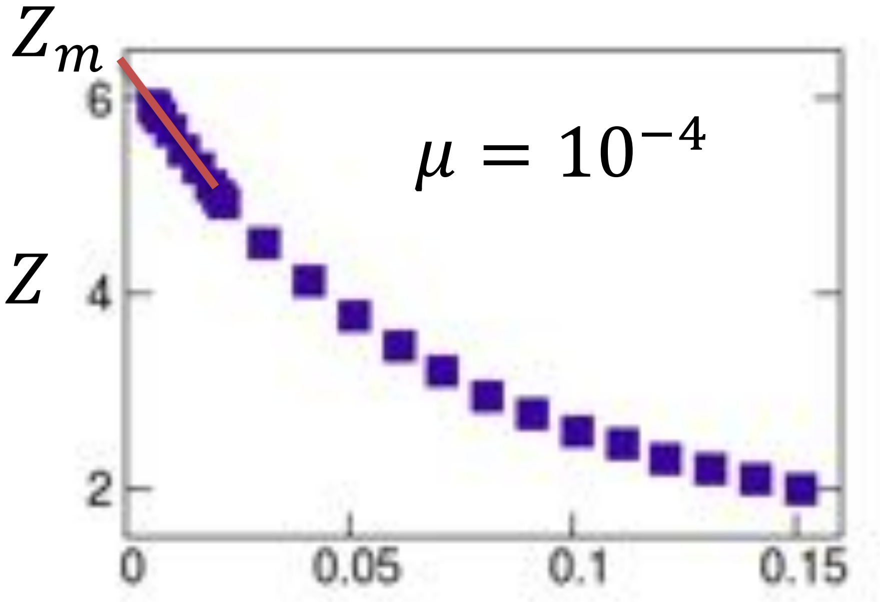
① ③ Assumed to be true for all μ [2]

[1] Lerner, During, and Wyart, PNAS 2012
[2] Wyart and Cates, PRL 2013

Coordination number deficit ΔZ for $\mu \to 0$

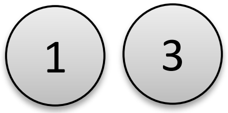


$Z = 2$

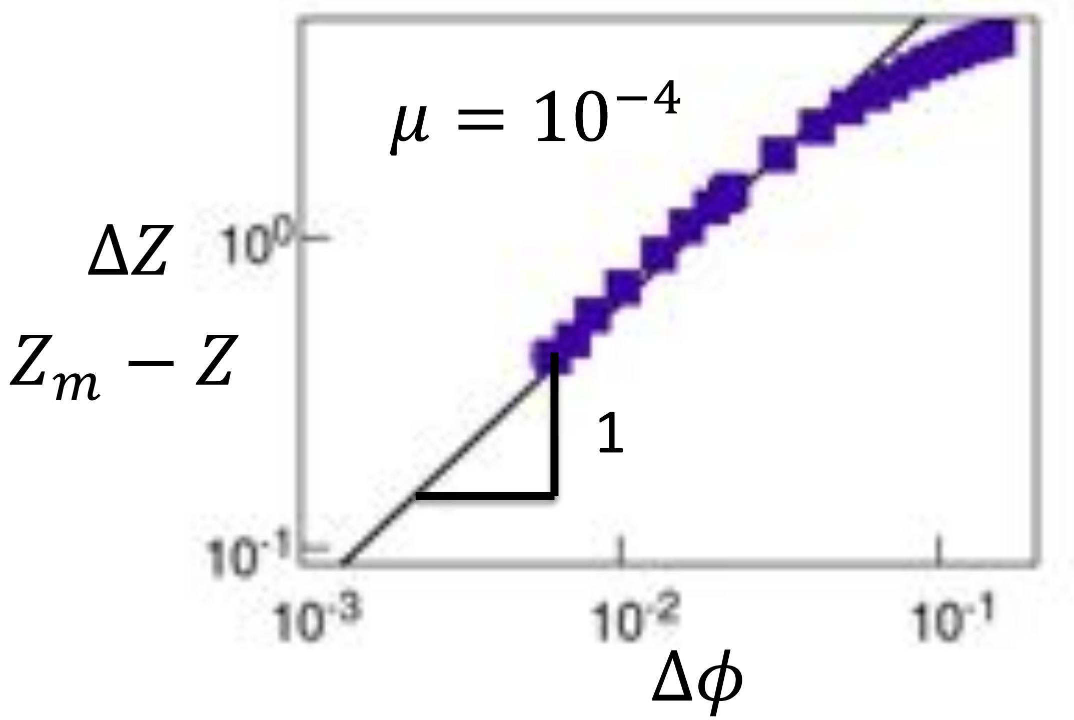


$\mu = 10^{-4}$

$\Delta\phi = \phi_m - \phi$



satisfied

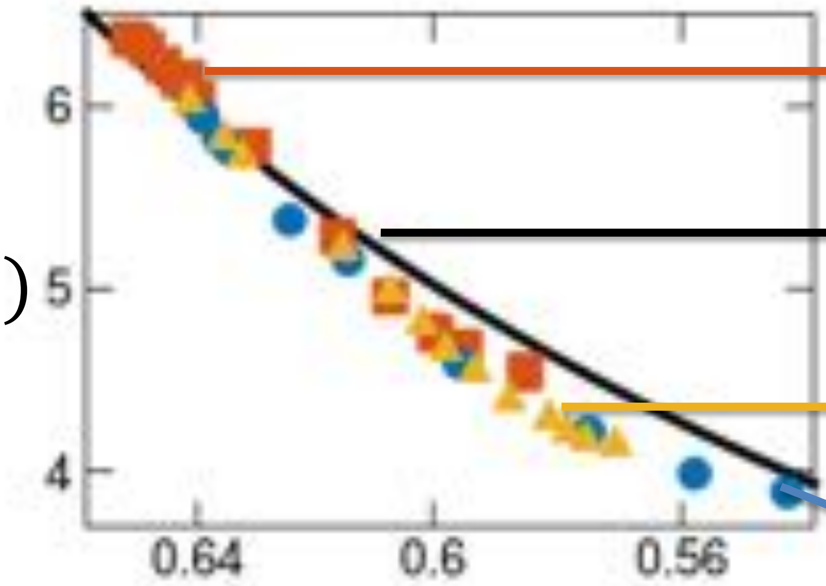


$\mu = 10^{-4}$

Lerner, During, and Wyart, PNAS 2012
Wyart and Cates, PRL 2013

Critical contact number and volume fraction

Critical contact number $Z_m(\mu)$



Critical volume fraction $\phi_m(\mu)$

Suspensions (Our results)

$$\phi_m = \frac{Z_m}{Z_m + 2\,a}$$ [1]

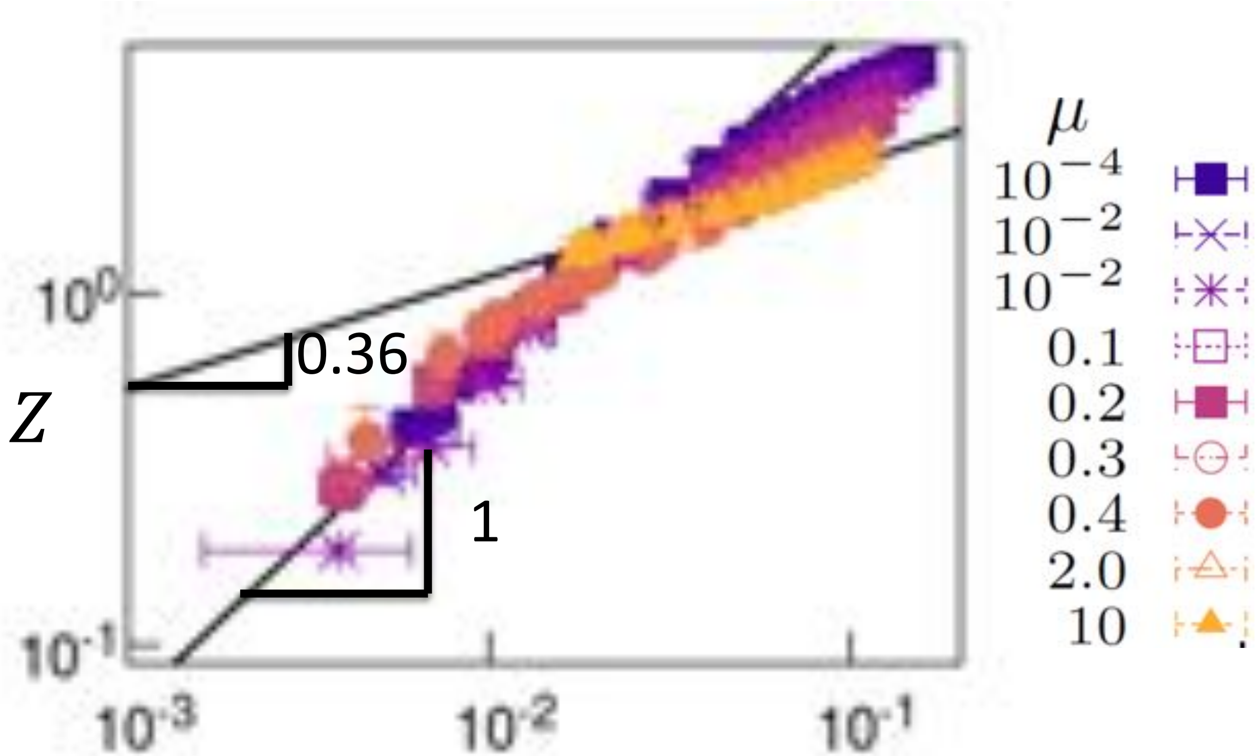
Sheared granular spheres [2]

Isotropic compressed packing of spheres [3]

[1] Edwards and Oakeshott, Physica A 1989, [2] Sun and Sunderesan J. Fluid Mech. 2011
[3] Silbert, Soft Matter 2010

Contact number deficit ΔZ vs. distance to jamming

$\Delta Z = Z_m - Z$

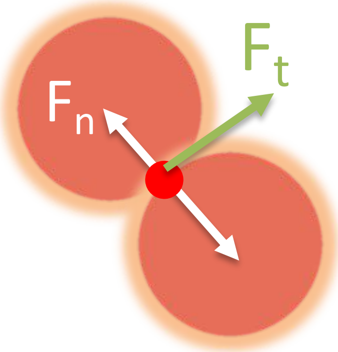


$\Delta\phi = \phi_m - \phi$

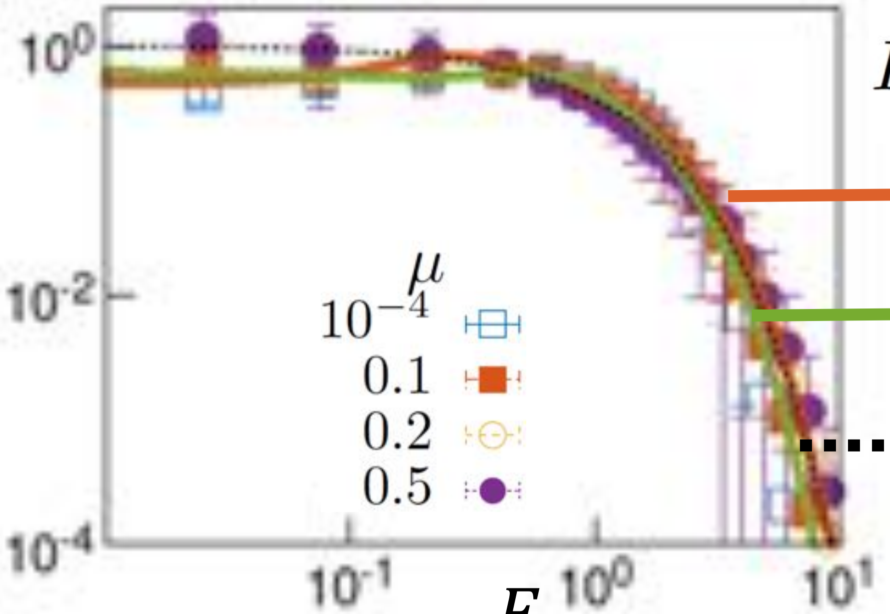
Wyart and Cates, PRL 2013

① ③ $(Z_m(\mu) - Z) \sim (\phi_m(\mu) - \phi)$ → Not true for all μ, and $\Delta\phi$

Normal contact force probability distribution $P(\theta)$



$$\phi_m - \phi \approx 0.005$$



$P(\theta)$

$$\theta = \frac{F_n}{\langle F_n \rangle}$$

$$P(\theta) = a\left(1 - be^{-c\theta^2}\right)e^{-\beta\theta}$$ [1,2]

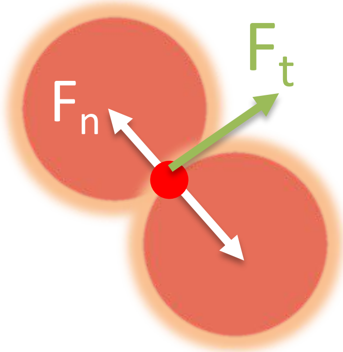
Fit for $\mu = 10^{-4}$

Packed amorphous spheres [2]

$a\exp(-\theta)$

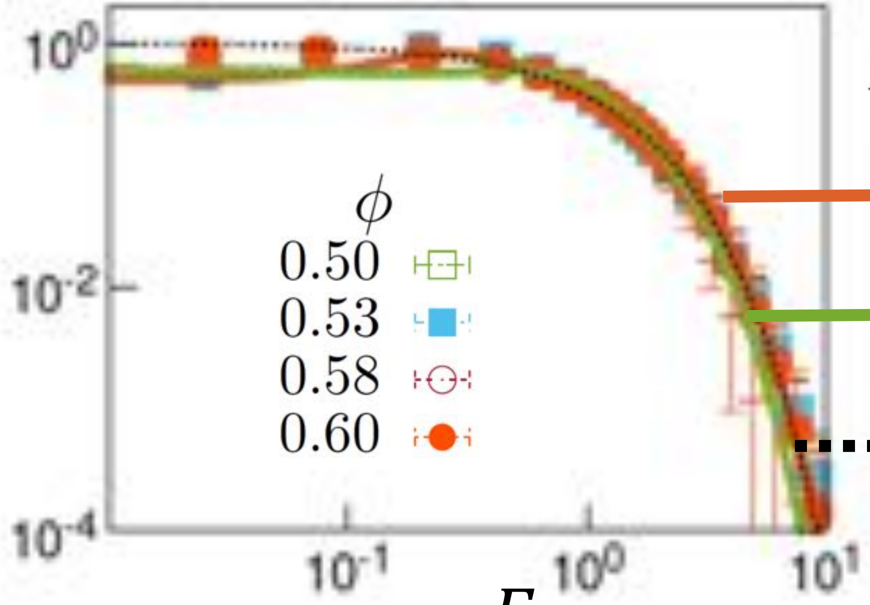
[1] Mueth, Jaegger and Nagel Phys. Rev. E 1998,
[2] Blair, Muggenberg, Marshall, Jaegger and Nagel Phys. Rev. E 2001

$P(\theta)$ for varying ϕ



$$\mu = 0.5$$

$P(\theta)$



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$$P(\theta) = a\left(1 - b e^{-c\theta^2}\right) e^{-\beta\theta}$$ [1,2]

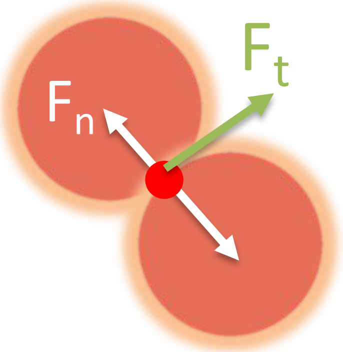
Fit for $\mu = 10^{-4}$

Packed amorphous spheres [2]

$a \exp(-\theta)$

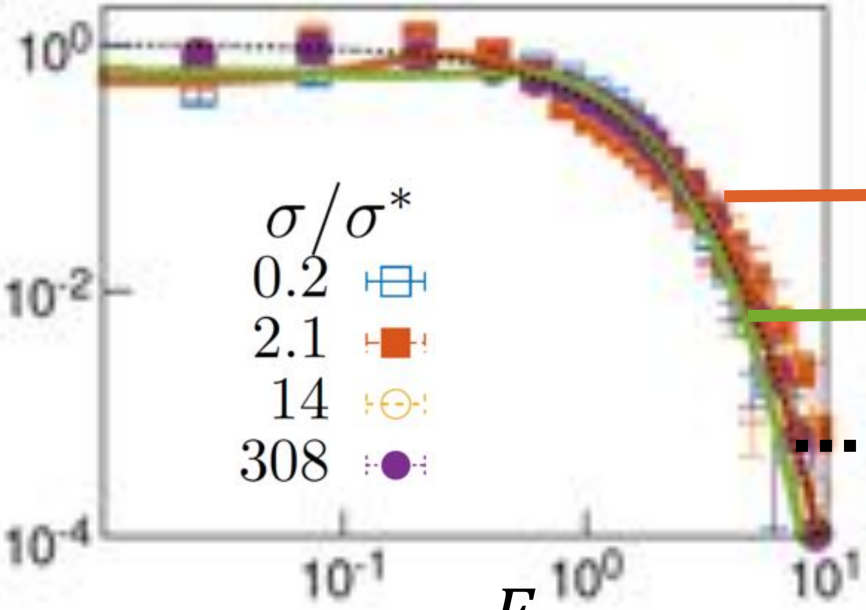
[1] Mueth, Jaegger and Nagel Phys. Rev. E 1998,
[2] Blair, Muggenberg, Marshall, Jaegger and Nagel Phys. Rev. E 2001

$P(\theta)$ for varying stress σ



$\mu = 0.5$

$P(\theta)$



$$\theta = \frac{F_n}{\langle F_n \rangle}$$

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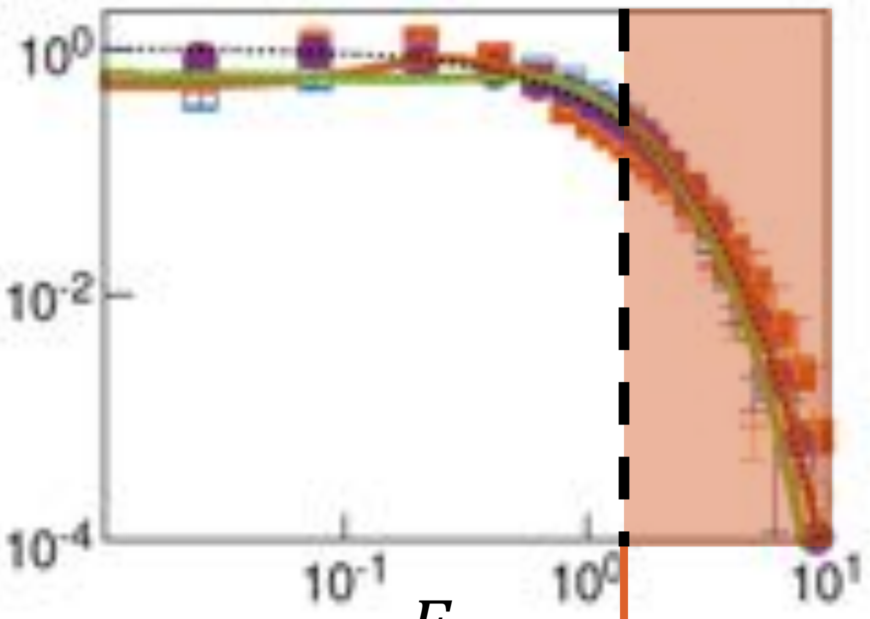
[1] Mueth, Jaegger and Nagel Phys. Rev. E 1998,

[2] Blair, Muggenberg, Marshall, Jaegger and Nagel Phys. Rev. E 2001

[3] Ness and Sun, Soft Matter 2016

Contact force distribution to frictional contacts

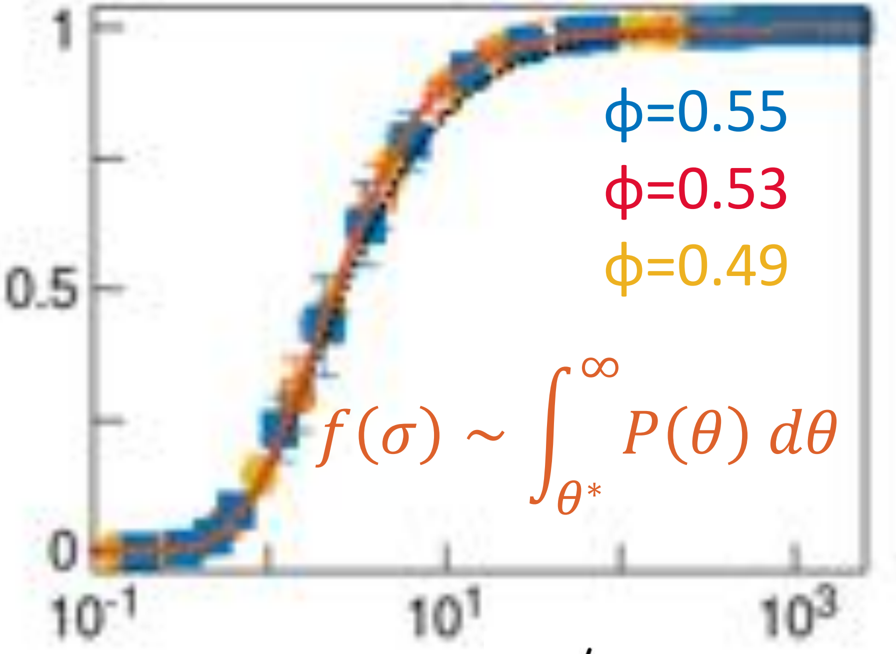
$P(\theta)$



$$\theta = \frac{F_n}{\langle F_n \rangle}$$

$$\theta^* = \frac{F^*}{\langle F_n(\sigma) \rangle}$$

$f(\sigma)$



ϕ=0.55

ϕ=0.53

ϕ=0.49

$$f(\sigma) \sim \int_{\theta^*}^{\infty} P(\theta)\, d\theta$$

Stress σ / σ^*

② Exponential tail of $P(\theta)$ gives rise to exponential $f(\sigma)$

Guy, Hermes and Poon PRL 2015

Conclusions

- $\phi_m(\mu), Z_m(\mu)$ and $P(F_n)$ of suspensions are similar to granular packings, and sheared granular materials
- ② Justified $f(\sigma) \sim \exp(-\sigma^*/\sigma)$ in the Wyart-Cates model
- ① ③ Contact number deficit, $Z_m(\mu)$ –Z, does not explain viscosity divergence and linear interpolation for the jamming volume fraction ϕ_J

Future Work

- Network properties: 3 cycles, efficiency etc. to understand the dynamics of shear thickening
- Use connections with granular materials to build better models

Thanks

Dense suspension team @ Edinburgh

Dr. J.P. Morrissey

Mr. Yang Cui

Rheology of non-Brownian suspensions composed of frictional and frictionless particles